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PG Semester I

Paper - CC2

Unit - 4

Topic - Subgroup

* Subgroup

A non-empty subset H of a group $(G, *)$ is said to be a subgroup of G , if $(H, *)$ is itself a group.

Note * Every group G has at least two subgroups, viz, $\{e\}$ and G itself.

These two subgroups are called trivial or improper subgroups.

* Any non-empty subset H of G is called a complex of the group G .

e.g.,

(i) $[\{1, -1\}, \cdot]$ is a subgroup of

$$[\{1, -1, i, -i\}, \cdot]$$

(ii) $(\mathbb{Z}, +)$ is a subgroup of $(\mathbb{Q}, +)$.

(iii) $(\mathbb{Q}, +)$ is a subgroup of $(\mathbb{R}, +)$.

(iv) The set of all non-singular matrices with real elements whose determinants are 1, is a subgroup of multiplicative group of all non-singular $n \times n$ matrices.

Criterion for a Subset to be a Subgroup

1. A non-empty subset H of a group G is a subgroup of G if and only if

(i) $a, b \in H \Rightarrow ab \in H$

(ii) $a \in H \Rightarrow a^{-1} \in H$

where a^{-1} is the inverse of $a \in G$.

2. A non-empty subset H of a finite group is a subgroup of G iff $ab \in H \forall a, b \in H$.

APRIL

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Note: In case of additive notation, the above results can be stated as a non-empty subset H of a group G is a subgroup iff

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$$(i) \quad a + b \in H \quad \forall a, b \in H$$

$$(ii) \quad -a \in H \quad \forall a \in H$$

which can be combined to a single condition $a - b \in H, \quad \forall a, b \in H$

(iii) Let H be a non-empty subset of a group G . Then H is a subgroup of G iff $a, b \in H \Rightarrow ab^{-1} \in H$, where b^{-1} is the inverse of b in G .